House Prices

This problem gives you the chance to:
• work with graphs and formulas in a real context

In March 2006, a newspaper article reported that houses in Maryland are so expensive that many people are unable to afford the monthly house payments.

This graph shows the average house price and the average monthly payment for all the different counties in Maryland.

House Prices and Payments
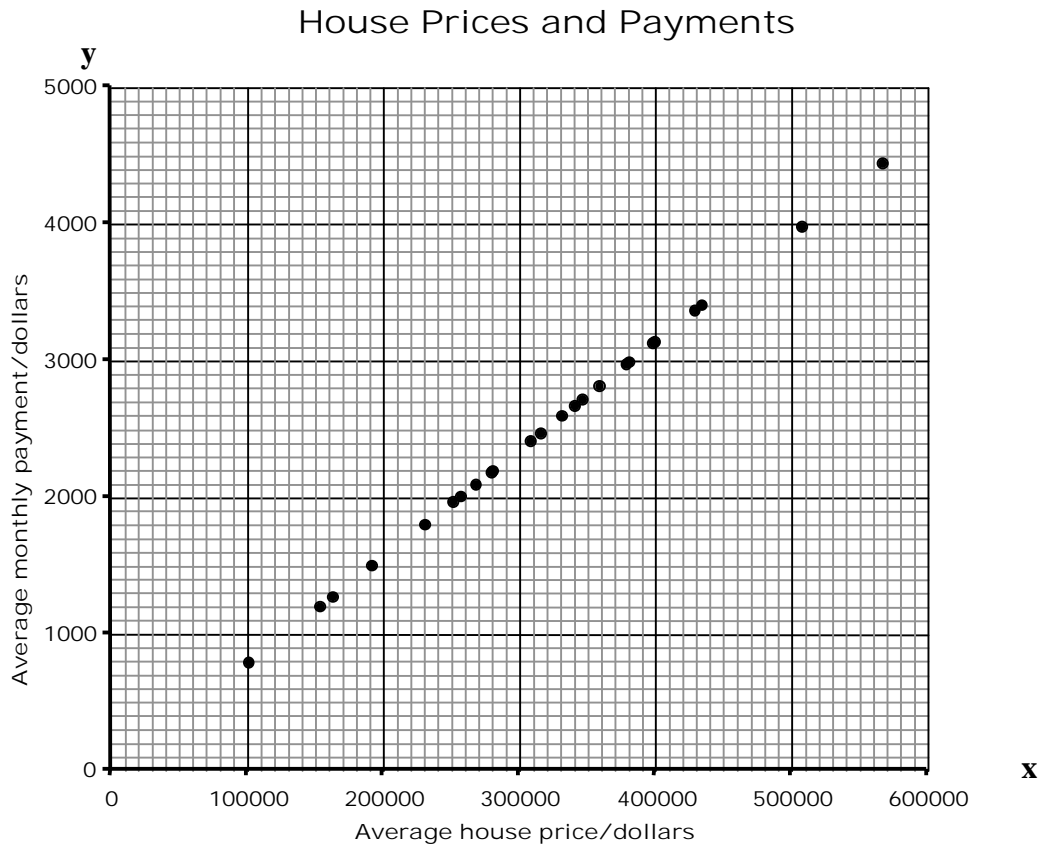
1.
   a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

   _____

   _____

   b. Find the monthly payment for a house costing $450 000.   _____

   c. Find a formula connecting the average monthly payment with the average house price.

   _____

This graph shows the average monthly wage and the average monthly house payment for each county in Maryland.

y   Wages and House Payments

2.
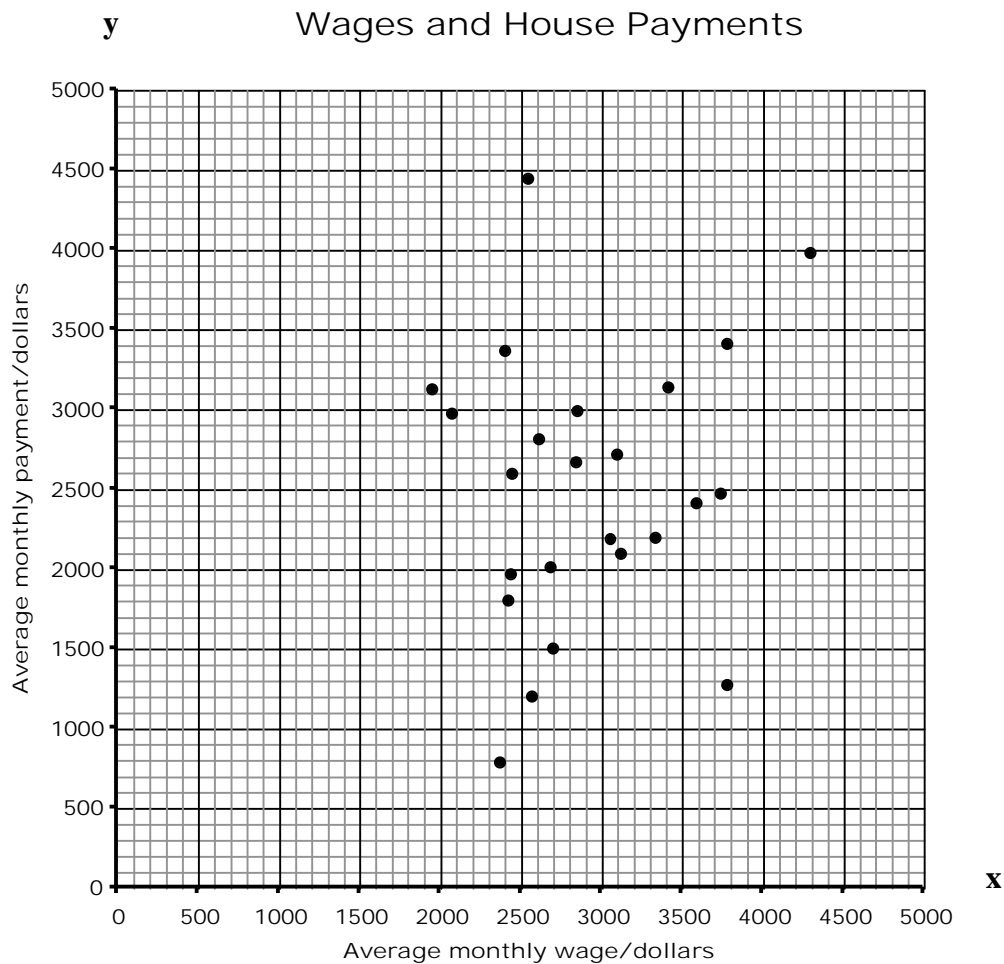
a. Describe the pattern of the data. _____

_____

b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.

_____

7

House Prices   Test 9

Task 2: House Prices		Rubric
The core elements of performance required by this task are: • work with graphs and formulas in a real context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1.a Gives correct explanation such as: There is a positive correlation between the two variables.	1	
b Gives correct answer in the range $3400 and $3800..	1	
c Gives correct answer such as: y = 0.008x (approximately) or y = x/125, where $x is the house price and $y is the monthly payment or equivalent Accept an intercept in the range 0 to 100.	1	3
2.a Gives correct explanation such as: No correlation or equivalent	1	
b Point A correctly indicated: (2500, 4450)	1	
c Point B correctly indicated: (2360, 800), or (3770, 1270)	1	
d Clear indication of correct region, above the line y = x	1	4
Total Points		**7**

House Prices

Work the task. Look at the rubric. What did you have to know mathematically to find the formula for connecting house prices and payments? To identify the area where the monthly payment is more than monthly income, what did you think about? What mathematics did you use?

Look at student responses for describing the graphs in 1a and 2a. Did your students have the mathematical vocabulary for discussing correlation and no correlation? What other types of descriptors did they use?
- Did they talk about the scale?
- Describe a pattern of going up?
- Did they try to give a numerical answer?
- Did they read information into the data, e.g. "the higher the wage the higher the payment"?

What types of experiences would help students to develop the language of reading and interpreting data and scatterplots? How could their descriptions be improved?

While the main purpose of the task is to look at data and work with formulas, the task also shows some problems students have with making sense of decimal and decimal place value. Look at student work for 1b. How many of your students:

≈ $3500	≈$4500	≈$3040 or $3050	≈$35,000 or $45,000	≈$350,000	Other

What are the implications for instruction?

Now look at student equations for 1c. Use my categories or make categories of your own. How many of your students:
- Could give the correct equation?
- Did not attempt this part of the task? Why do you think so many students were unwilling to attempt this part of the task?
- Gave a ratio instead of using algebraic notation, e.g.
  - $100,000 house price equals $800 in house payments or
  - for every $100000 divide by $100
- Used a third variable: x=y/sy, x/y times x = payment, ym=x
- Used the formula for a line, y=mx+b
- Made a decimal error: e.g. x=4/5y or y=.8x
- Tried to find a set difference: 100,000 – 10,000, 100,000 – 1000, p = h - 100,000, p = h - 10,000

What other types of errors did you see?

Look at student work for identifying the points on the graph. Students made the most errors when finding the point where it will be most difficult to make the payment. How many of your students put

(2500,4500)	(1900,3100)	(4300,4000)	Circled a region	Marked points with no labels	Didn't attempt

What opportunities do students have to make sense of the meaning of points on a graph, i.e. find a point based on context and meaning rather than from a numerical reference? What are some of the tasks you could use with students to help promote this type of thinking?

The most difficult part of the task for students was 2d, indicate the part of the graph where the average monthly house payment is more than the average monthly wage. Have your students worked with graphing inequalities?

How many of your students:
- Drew a line to show where average wage equals monthly payment?
- Circled a few correct points, but not all?
- Gave a description that only fit point A?
- Said the area in the middle? At the top? At the left? At the right?
- Gave a range without mentioning axes or labels: e.g. between 2000 and 3000 or over 2500?
- Did not attempt this part of the task?

What do you want students to understand about inequalities? When working with inequalities are the problems presented in a context? Do you have any favorite tasks to help students make sense of inequalities? What further experiences do students need?

Looking at Student Work on House Prices

Student A is comfortable using the graph as a tool and draws lines on both graphs to help make sense of the situation. While the student has one of the top scores on this task, the student does not have the formal vocabulary for talking about data, such as correlation or no correlation. The student drew in the line for wage equaling payment, but did not clarify which part of the graph represents the inequality payment greater than wage.

Student A

This graph shows the average house price and the average monthly payment for all the different counties in Maryland.

House Prices and Payments
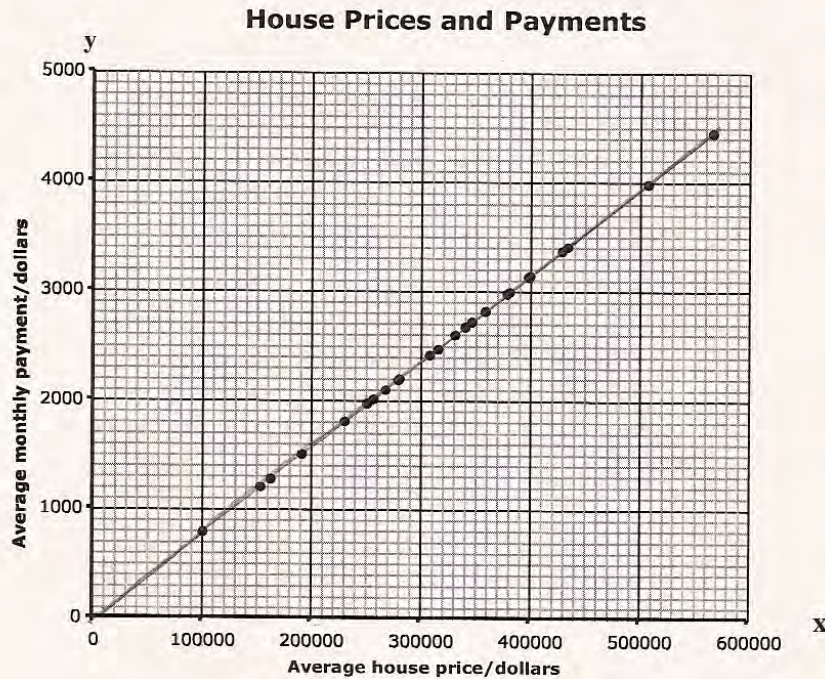
1.
   a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

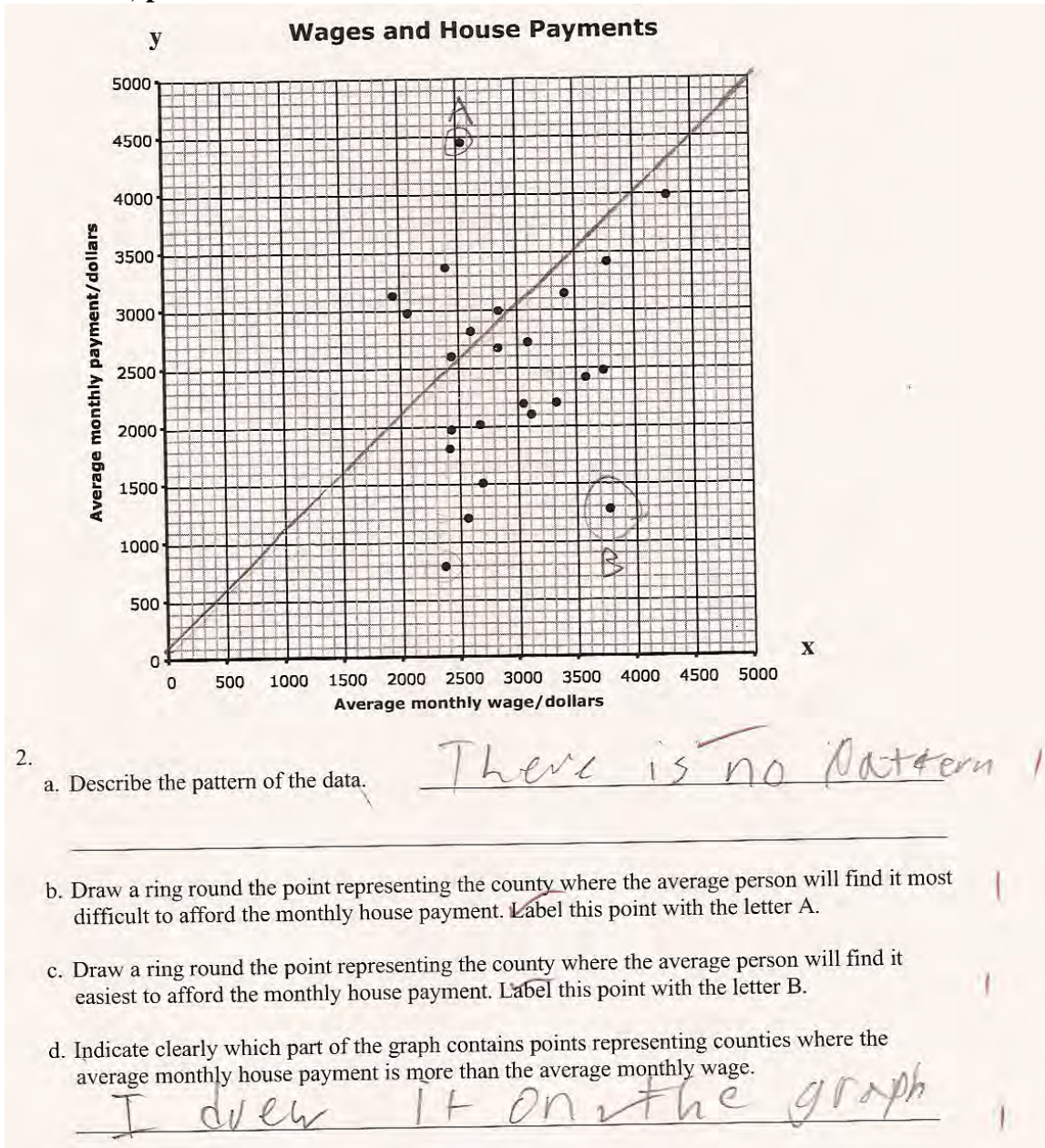   _The higher the house price the higher the monthly payment_

   b. Find the monthly payment for a house costing $450 000. ✓ _3,500_

   c. Find a formula connecting the average monthly payment with the average house price.

   _y = x/125_

Student A, part 2

Wages and House Payments

2.
   a. Describe the pattern of the data. _There is no pattern_

   _____

   b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

   c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

   d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.
   I drew It on the graph

Student B also lacks the formal vocabulary for discussing data on a scatterplot. The student draws lines to show the specific points currently on the graph where the payment is higher than the wage, but the lines don't define the entire area of the graph where this is true. So the student is making sense of context, but not reasoning about the domain of the solution.

Student B
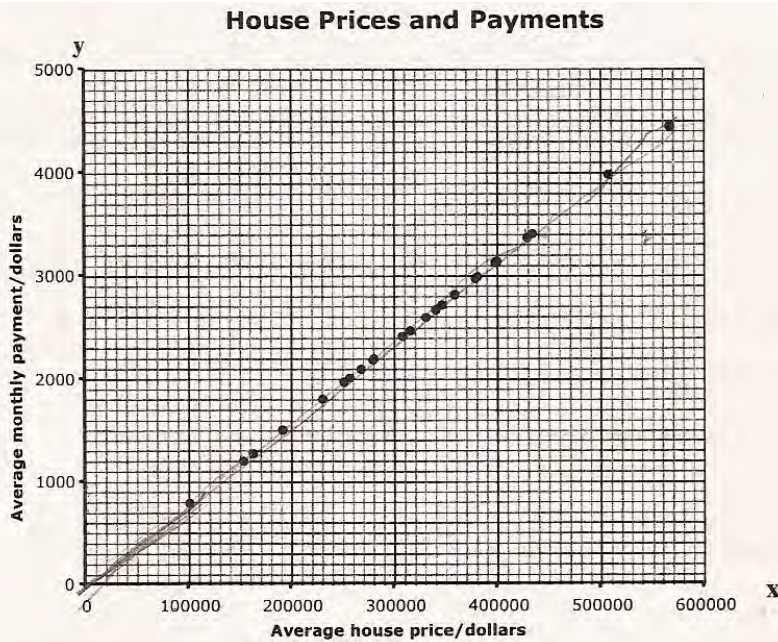
House Prices and Payments

1.
   a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

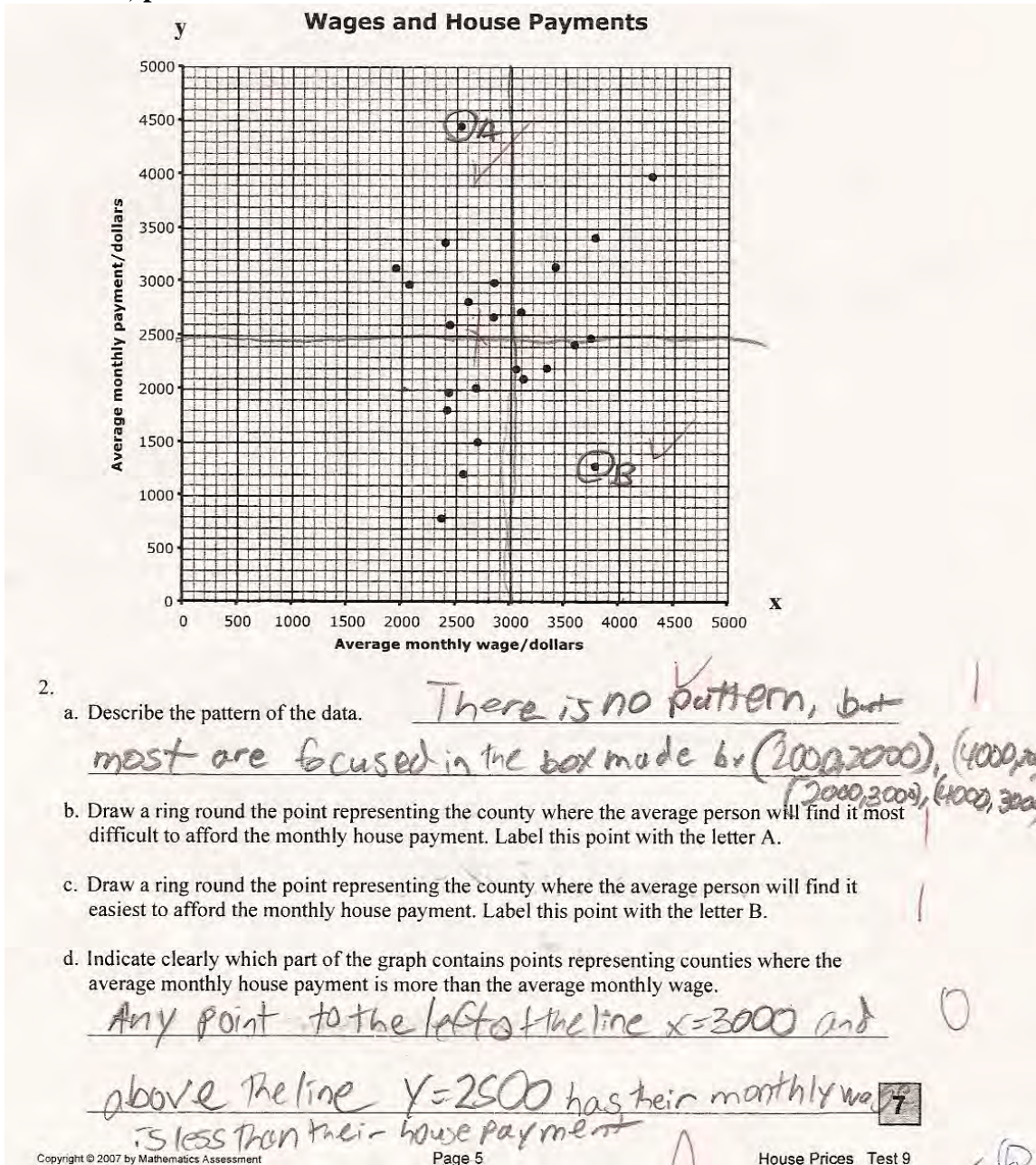   This indicates that as average house prices go up, so do the average monthly payment, and that Y=.008x

   b. Find the monthly payment for a house costing $450 000. ___3600 ✓___

   c. Find a formula connecting the average monthly payment with the average house price.

   Average monthly payment = .008 (average house price)

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Student B, part 2

Wages and House Payments

2.
   a. Describe the pattern of the data. _There is no pattern, but most are focused in the box made by (2000,2000), (4000,2000), (2000,3000), (4000,3000)_

   b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

   c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

   d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage. _Any point to the left of the line x=3000 and above the line Y=2500 has their monthly wage is less than their house payment_

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Student C, representing about 38% of the students, did not attempt to make a formula in part 1c. In thinking about the graph, the student makes a logical conclusion that is not supported by the data. In part 2d, the student identifies two points that meet the constraints, but does not define the entire region represented on the graph for payments greater than wage.

Student C

1.
   a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

   the monthly payments increase when the house prices increase. ✓   1

   b. Find the monthly payment for a house costing $450 000.   _around $3500 ✓_   1

   c. Find a formula connecting the average monthly payment with the average house price.

   _✗_   0

Wages and House Payments
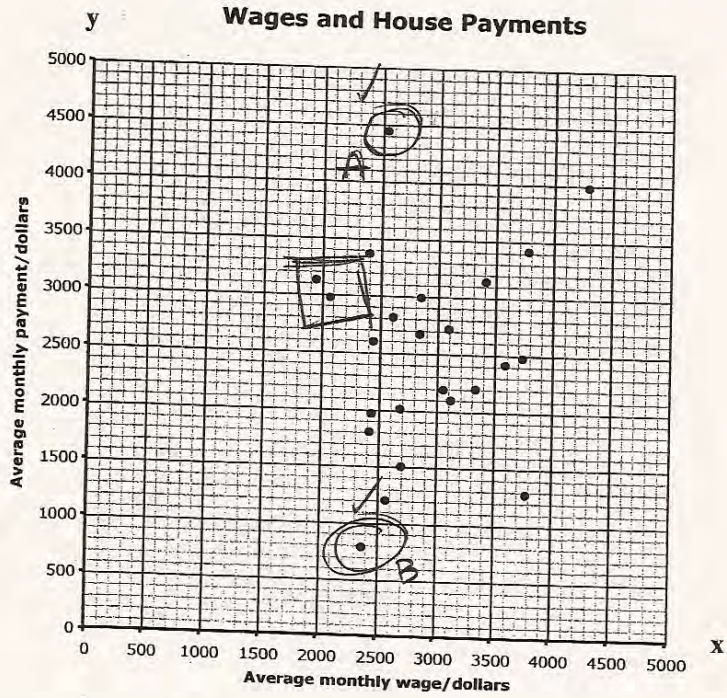
2.
   a. Describe the pattern of the data.   _the average monthly house payment ✗ increases when your average monthly wage increases._

   b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

   c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

   d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.

   i put a square around those points. points around ✗ the upper left corner are the ones that represent counties where house payments are higher than monthly wage.   7

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Student D does not talk about the trend in the data in part 1a, but instead describes the scale. In attempting the formula, the student gives a specific value (if the house costs 300,000 then the payment is 2,500) reading a value from the graph. The student is using the variable as a label, the equation really reads the house payment is $2500 for a house which equals or costs $300,000. In part 2a the student is giving an opinion rather than finding a trend in the data. In 2d, the student identifies discrete points on the graph: one true, one false, and one where the values are equal.

Student D

1.
a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

On the monthly prices they add 1000 on the ✗

save wages they add w,000.

b. Find the monthly payment for a house costing $450 000.   _$3,500_ ✓

c. Find a formula connecting the average monthly payment with the average house price.

_2,500 × = 30,000_   ✗

Wages and House Payments

2.
a. Describe the pattern of the data.   _1000 and share you have to pay_ ✗

a lot of wages and a lot of monthly payments

b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.

_(2500, 2000) (1500, 1500) (1500, 2500)_   ✗

Student E makes an attempt to find the relationship between payment and house price. The student makes a table of values, but is possibly relying on number sense to equate the two values rather than knowing how to calculate a value. *Are students given enough opportunities to find relationships and patterns that are not based on whole-number values?* In part 2d the student identifies one point that meets the condition of payment exceeding wage, but does not think about the possibility of more solutions. *How can we help students build the internal dialog of asking questions, such as "Is this the only case when this is true? What tools might help me find all the possibilities?"*

Student E

Average house price/dollars

1.

a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

The higher the average house price is, the higher the average monthly payment is. (example: FOR a House 100000, the monthly payment will be 800. 300,000 AVG. House PRICE = more monthly payment) ✓

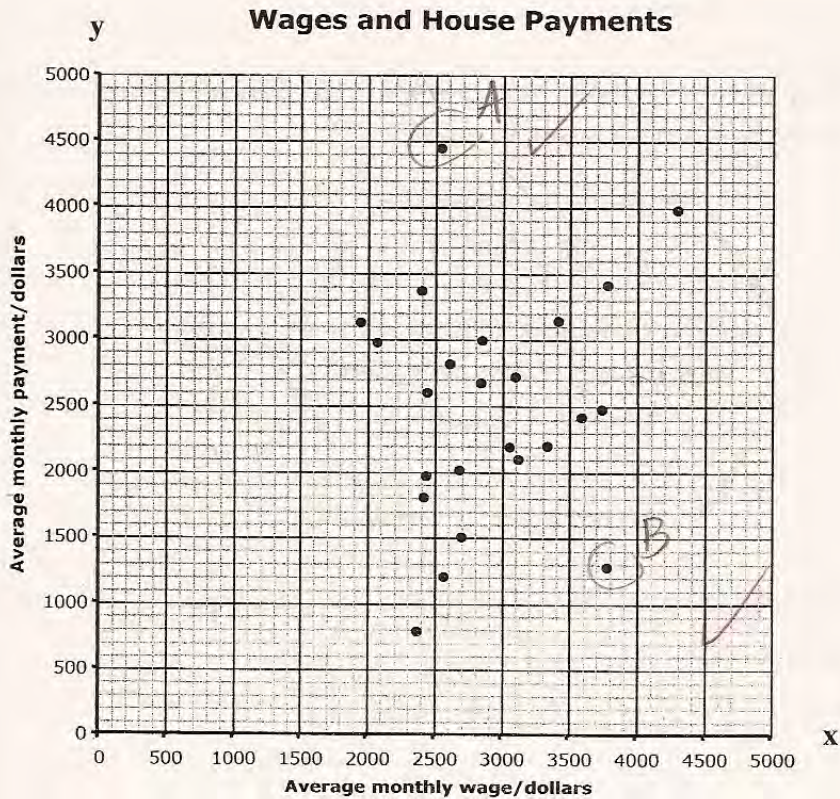b. Find the monthly payment for a house costing $450 000.   3,525 ✓

Avg. Monthly Payment = M

AVG. HOUSE PRICE = H

c. Find a formula connecting the average monthly payment with the average house price.

230,000 = 1,800   430,000 = 3,900
330,000 = 2600   530,000 = 4,200   250,000 = 1975
380,000 = 3,000

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Student E, part 2

Wages and House Payments

a. Describe the pattern of the data.   The higher up & left the point is, the
more difficult it is for the citizen to pay. Going to opposite away, it's
easier for them to pay. (down & right). There's no pattern other than that

b. Draw a ring round the point representing the county where the average person will find it most
difficult to afford the monthly house payment. Label this point with the letter A.

c. Draw a ring round the point representing the county where the average person will find it
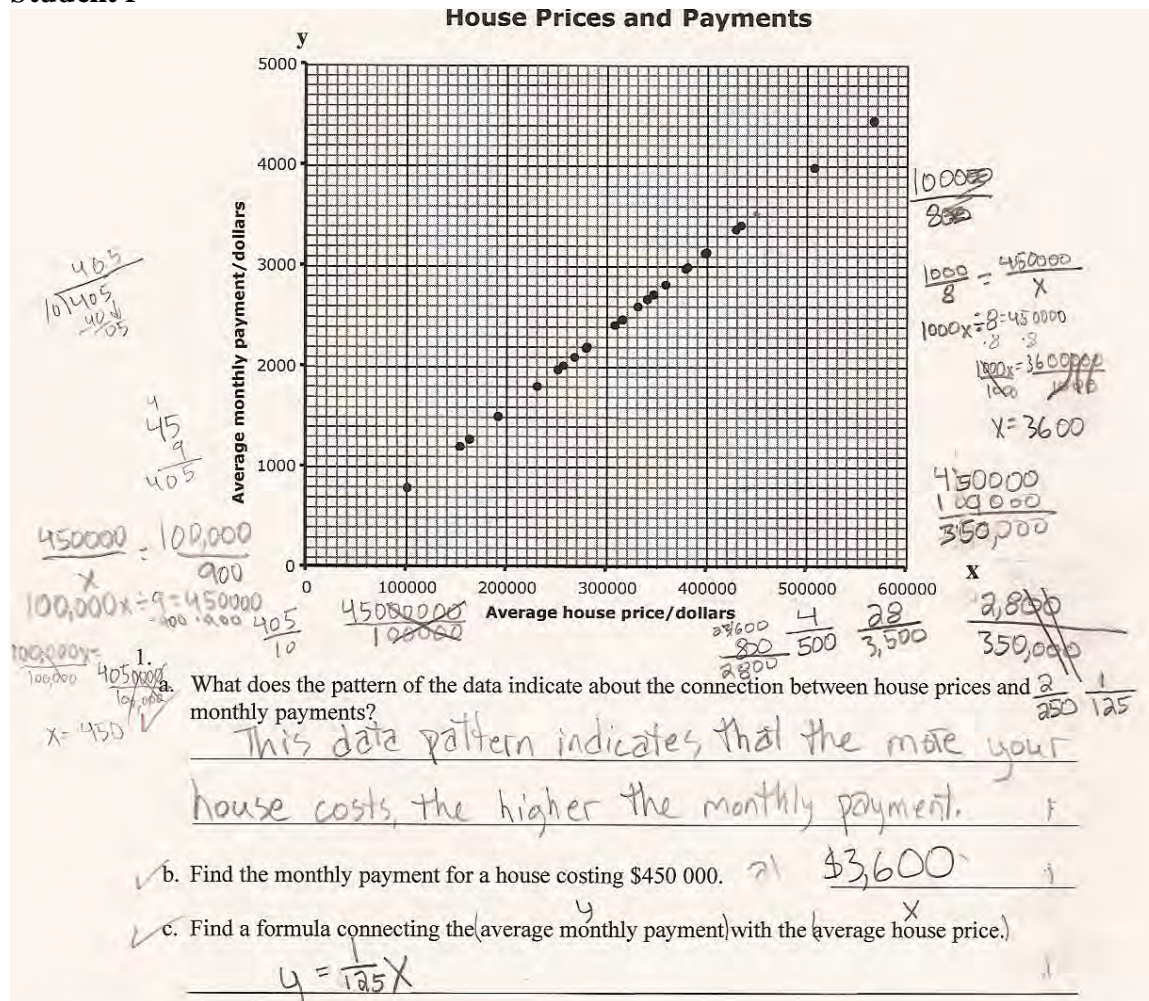easiest to afford the monthly house payment. Label this point with the letter B.

d. Indicate clearly which part of the graph contains points representing counties where the
average monthly house payment is more than the average monthly wage.

In the circle A, the payment is more than the
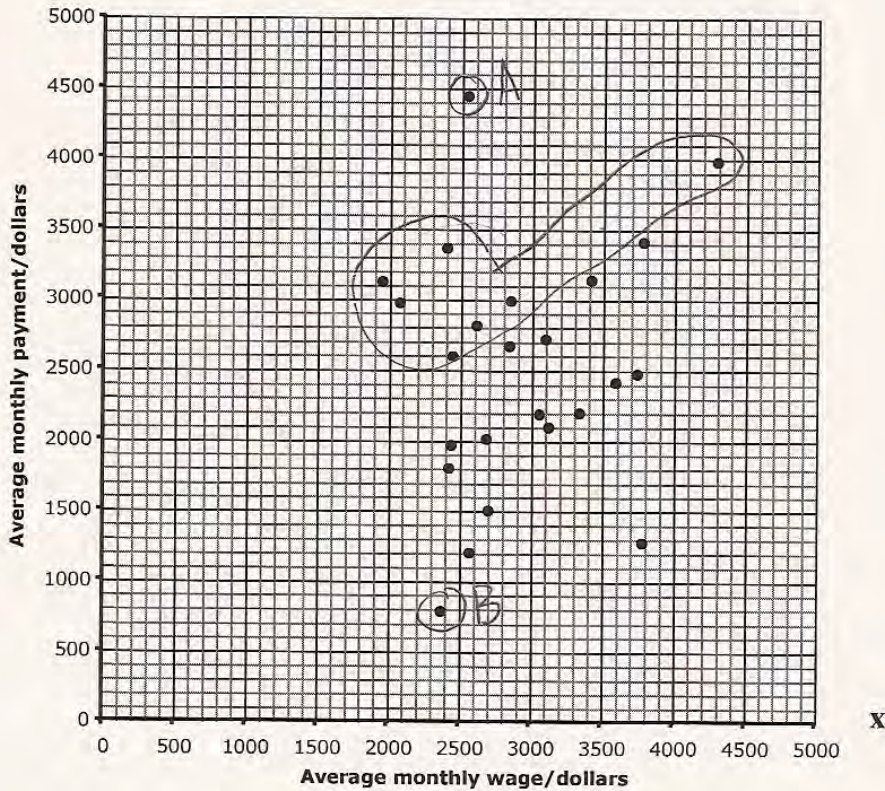
average monthly wage.   ⋀

7

Finding the relationship between average monthly payment was truly a problem-solving struggle for students. The work of Student F shows the amount of investigating the student needed to do solve the problem. *How would you categorize the strategies used? Which calculation finally helped the student with the solution? What algebraic tools would you want the student to use instead?* In part 2d the student finds some of the points that meet the criteria, but includes one that does not. The student does not use the information about wage equaling monthly payment to help define the area for the inequality.

Student F

House Prices and Payments

a. What does the pattern of the data indicate about the connection between house prices and monthly payments?

This data pattern indicates that the more your house costs, the higher the monthly payment.

b. Find the monthly payment for a house costing $450 000. $3,600

c. Find a formula connecting the (average monthly payment) with the (average house price.)

$$y = \frac{1}{125}X$$

Student F, part 2

y

Wages and House Payments

a. Describe the pattern of the data. There is no pattern in this collection of data.

b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.

All the points in the large circle represent counties where the average monthly house payment is more than the average monthly wage. They form a shape like the Big Dipper.

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Algebra   **Task 2**   **House Prices**

Student Task	Work with graphs and formulas in a real context.
Core Idea 5 Data Analysis	**Select and use appropriate statistical methods to analyze data.** • Understand the relationship between two sets of data, display such data in a scatterplot, and describe trends and shape of the plot including correlations (positive, negative, and no) and lines of best fit. • Make inferences based on the data and evaluate the validity of conclusions drawn.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Use symbolic expressions to represent relationships arising from various contexts. • Approximate and interpret rates of change, from graphic and numeric data.

Mathematic in this task:
- Understanding information on a scatterplot, looking for trends such as correlation or no correlation
- Recognizing that a linear function passing through the origin is a proportion and finding a formula for a proportion
- Reading and interpreting points on a graph
- Graphing inequalities

Based on teacher observations, this is what algebra students knew and were able to do:
- Read and locate points on a scatterplot to meet constraints of the context
- Recognize when there is no pattern in a scatterplot
- Describe a trend in a scatterplot

Areas of difficulty for algebra students:
- Finding a formula for a line on a graph
- Graphing an inequality on a graph from a verbal description
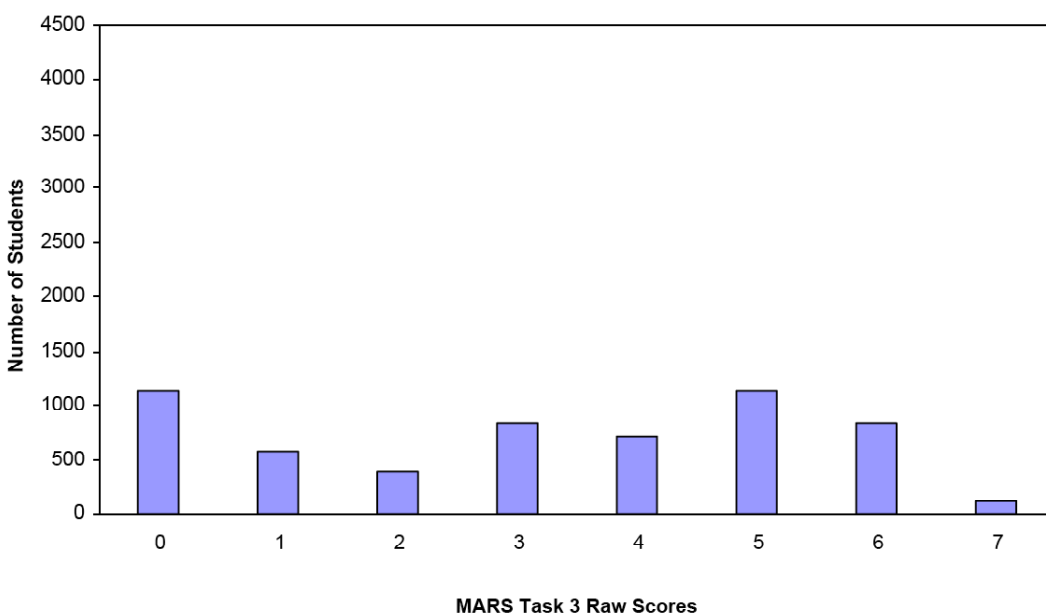
Task 2 - House Prices

Mean: 3.50   StdDev: 1.75

Table 46: Frequency Distribution of MARS Test Task 2, Course 1

Task 2 Scores	Student Count	% at or below	% at or above
0	400	7.0%	100.0%
1	445	14.7%	93.0%
2	766	28.0%	85.3%
3	1004	45.5%	72.0%
4	1284	67.9%	54.5%
5	1208	88.9%	32.1%
6	495	97.5%	11.1%
7	142	100.0%	2.5%

Figure 56: Bar Graph of MARS Test Task 3 Raw Scores, Course 1

MARS Task 3 Raw Scores

The maximum score available for this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, 93%, could either find the monthly payment from the graph or could state that there was no pattern to the second scatterplot. About half the students, 55%, could explain the trend of the first graph, find the monthly payment, and locate the two points on the second graph for easiest and most difficult to afford. Some students, about 32%, could also describe the randomness of the second scatterplot. A few students, about 11% could also write a formula for a line on graph. Only 2.5% of the students could meet all the demands of the task including graphing and labeling an inequality on a graph. 7% of the students scored no points on the task. 75% of the students with this score attempted the task.

House Prices

Points	Understandings	Misunderstandings
0	75% of the students with this score attempted the task.	Students could not locate the y coordinate on the graph. Students had difficulty with decimal errors, giving an answer in the ten thousands (6%), hundred thousands (6%), or even hundreds. Another decimal error was understanding what is halfway between 3000 and 4000. 4% of the students put numbers such as 3050 for 3500.
1	Students could find the monthly payment or notice that the pattern on graph 2 was random	About 6% of the students made a false statement about the graph, "The higher the wage the higher the payment." About 9% said it increases or it decreases.
4	Students could state the trend for the first graph, find the payment on the graph, and locate the points for easiest and hardest to afford.	13% of the students just said that the graph goes up for the first graph. For point A the two most common errors were (1900,3100) and (4300,4000). 8% of the students marked a region for the points instead of a specific point. 5% marked two points, but didn't label them A and B.
5	Students in this group could also identify the lack of pattern or randomness of the second graph.	Students struggled with writing a formula. 38% did not even attempt a formula. About 4% gave a formula with a decimal error (e.g. y=.8 x instead of y = .008 x). Some students added a third variable (ym=x; y/x=paymt). Some students tried to give a rule around place value (100,00 – 10,000; h- 100,000; take off 2 zeroes; y=x/100) Others gave ratios (for every 100,000 its 100;) Many wrote y=mx+b, with no attempt to give values for m and b.
6	Students could write an equation for the proportion in part 1.	Students could not graph an inequality. 16% did not attempt to find the area for payment larger than wage. 16% circled a few correct points, but not all. 8% just picked point A. 10% said the middle, 6% left side, 10% the top, 3% the right. 16% gave a range, such as 2000 to 3000, with no labels or reference to the other variable.
7	Students could meet all the demands of the task: read and locate points on graph, describe a trend in scatterplots, write a formula for the equation of a line on a graph, and graph an inequality.	

Implications for Instructions

Students need opportunities to work with data in context in a variety of formats. Students need to have conversations about the data to help them build the academic vocabulary of discussing data, such as understanding correlation and no correlation. Students should be able to apply the skills of writing an equation for a line to practical situations, such as finding the formula for a line on a data graph.

Students at this grade level should be encouraged to make and test conjectures. Students should also develop the habit of mind to use points on the graph to verify the validity of their formulas. No student in the sample showed evidence of checking their formula against points on the graph. A quick test would show that 100,000 – 10,000 does not equal the payment of $800.

Many of the students struggled with decimal place value and calculations with decimals. They might benefit from a two or three week separate class to review and understand decimals. Some schools do this as an after school class or Saturday class. Some schools might have a second math class to work on mathematical concepts to support weaker students placed in Algebra.

Action Research – What Would It Take?

Sit down with colleagues and discuss some of the gaps in students' mathematical knowledge when they come to algebra. Do they have weaknesses in working with decimals, understanding fractions, understanding number operations, mathematical literacy and working with word problems, percents, and measurement? How can you design a program to support these students?

Are these gaps reasonable to fill within the constraints of the regular program? How much time might it take to work with a small group of students to diagnose and give them the information they need? How is the type of instruction they need different from the instruction in teaching a concept for the first time? What does it take to help students confront a misconception and replace it with a different idea?

Pick one small topic or problem area that everyone agrees is problematic for students. Find a variety of lessons from the materials up for adoption on the intervention list. Divide up the lessons and have each teacher try a set in their classroom or with a small group of students in a pull out program during prep or a few days of an after school class. Which materials seemed to have the most promising results? Now have all teachers try those materials, for that one topic. If possible try to video tape the sessions or arrange to view the lessons during each other's preps. What would it take to really implement one of those programs? What would be the pros and cons? What kind of professional development would be needed by the staff to assist in implementation? How can the work be split between all math teachers to allow for discussion and collaboration to make this effective and a team effort? Would it be possible to build in some common prep periods for planning?

Performance Assessment Task
House Prices Grade 9 task aligns in part to CCSSM grade 8
The task challenges a student to demonstrate understanding of the concept of scatter plots. A student must understand the relationship between two sets of data, be able to display such data in a scatter plot, and describe trends and shape of the plot including correlations (positive, negative, and no) and lines of best fit.   A student must make sense of scatter plots in order to make inferences on the data and evaluate the validity of the conclusions they draw. A student must approximate and interpret rates of change from graphic and numeric data to determine a rule or formula.
Common Core State Standards Math - Content Standards
Statistics and Probability **Investigate patterns of association in bivariate data.** 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **Functions** **Define, evaluate, and compare functions.** 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **Use functions to model relationships between quantities.** 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP. 2 Reason abstractly and quantitatively**. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative

reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2007	9	4	55%